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The thermal expansion of carbon fibre reinforced plastics

Part 4 *Ply multidirectional effects*

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Measurements of the principal linear thermal expansion coefficients of a tridirectional (−45°, 0°, +45°) carbon fibre reinforced plastics laminate are reported in the approximate temperature range 90 K to 500 K. A quantitative evaluation of the in-plane results in thermoelastic terms has yielded an agreement with these results consistent with the approximations and uncertainties involved. The qualitative agreement with expectations based upon the behaviour of unidirectional and bidirectional laminates is also demonstrated. The account concludes with an examination of some effects which are peculiar to multidirectional laminates.

1. Introduction

Earlier papers in this series describing the results of investigations of the linear thermal expansion coefficients of carbon fibre reinforced plastics have dealt with the specific influence of fibre type and orientation [1], fibre volume fraction [2] and resin matrix [3]. The measurements have served to establish the characteristic features of the temperature dependence of the thermal expansion of test specimens which are representative of the basic components from which practical structures are commonly built. In this survey it was logical to investigate next the thermal expansion characteristics of a tridirectional composite, in the evaluation of which use could be made of the knowledge gained earlier. The account which follows contains a description of this investigation, together with an objective assessment of the results within a pattern which includes some of the earlier results as well as an appraisal in fundamental terms.

Attention is finally turned to two subsidiary topics which are peculiar to angle-ply laminates: (a) the temperature dependence of inter-ply angles and (b) the influence of edge effects upon the thermal expansion characteristics.

2. The specimens and their investigation

Principally because of its interest to the aerospace industry, Code 69 resin‡ was chosen as the matrix, and in order to preserve the standardization of fibre type and origin as far as possible, Courtaulds HTS carbon fibre was chosen for the reinforcement. The fibre employed on this occasion was Grafil HTS 130 SC/10000, the Young's modulus of which is a few per cent lower than that of the HTS fibre Batch PT 112/21 Z employed earlier. In aiming to produce a construction which combined mechanical strength with a very low linear thermal expansion coefficient in one direction a −45°, 0°, +45° ply sequence was adopted. To be geometrically balanced, the bar was laid up from exactly similar sheets of pre-impregnated fibre in the orientation sequence:

0°, −45°, +45°, 0°, −45°, +45°, 0°, −45°, +45°,
0°, −45°, +45°, 0°, +45°, −45°, 0°, +45°, −45°,
0°, +45°, −45°, 0°, +45°, −45°, 0°.

In order to weight the contributions to the thermal expansion from the three directions equally, half of the outer ply layer on each side of the bar was removed by grinding after manufacture. De-

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TABLE I The specimens

Bar number	Specimen designation	Fibre orientations	Direction of thermal expansion measurements	Fibre volume (%)	Void content (%)
17	35	−45°, 0°, +45°	0°, in plane of laminate	56.0	0.7
17	36	−45°, 0°, +45°	90°, in plane of laminate	56.0	0.7
17	37	−45°, 0°, +45°	Perpendicular to plane of laminate	56.0	0.7

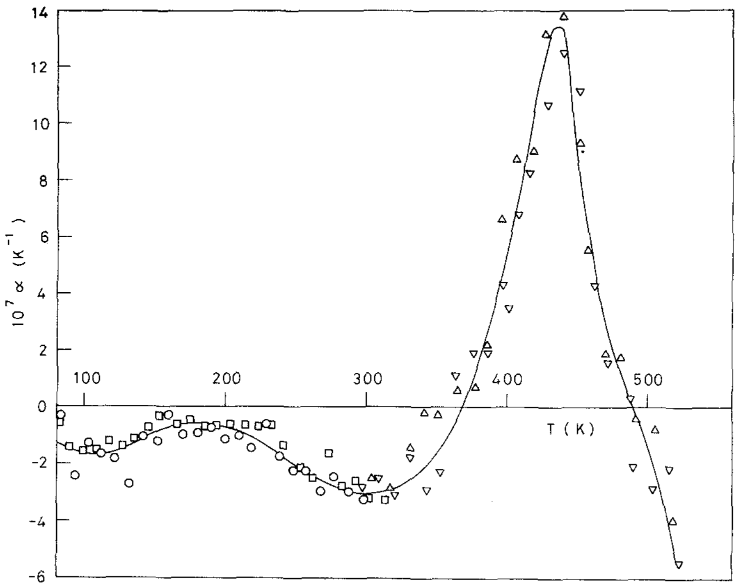

Figure 1 The linear thermal expansion coefficient α of specimens 35: ○ run 1; □ run 2; △ run 3; ▽ run 4.

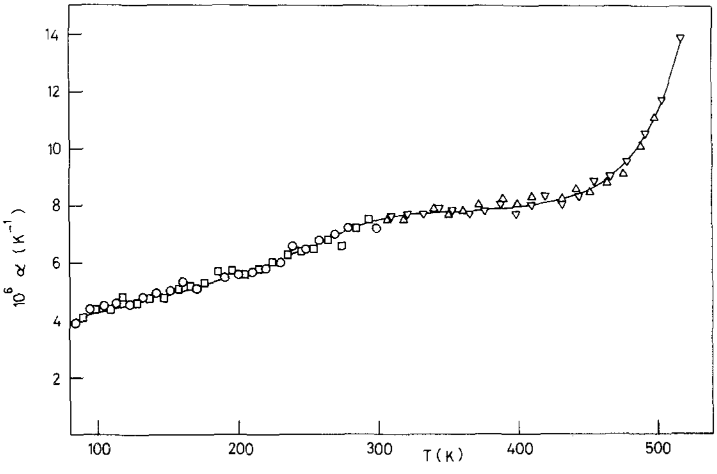

Figure 2 The linear thermal expansion coefficient α of specimens 36: ○ run 1; □ run 2; △ run 3; ▽ run 4.

tails of the preparation and curing cycle of the bar and the specimens prepared from it follow those described earlier for bars 13, 14 and 15 [3]. Preserving and extending the sequence of numbering employed in the earlier accounts, the designations of the subjects of the investigations are summarized in Table I; direct reference is also made to the graphs and specimen designations given previously [1–3].

3. Results

The thermal expansion was studied interferometrically by the experimental procedure described earlier [1]. Figs. 1 to 3 illustrate the primary data in graphical form in order to convey some idea of the experimental uncertainties involved and for comparative purposes; smoothed data are collected in Table II for analytical and reference purposes.

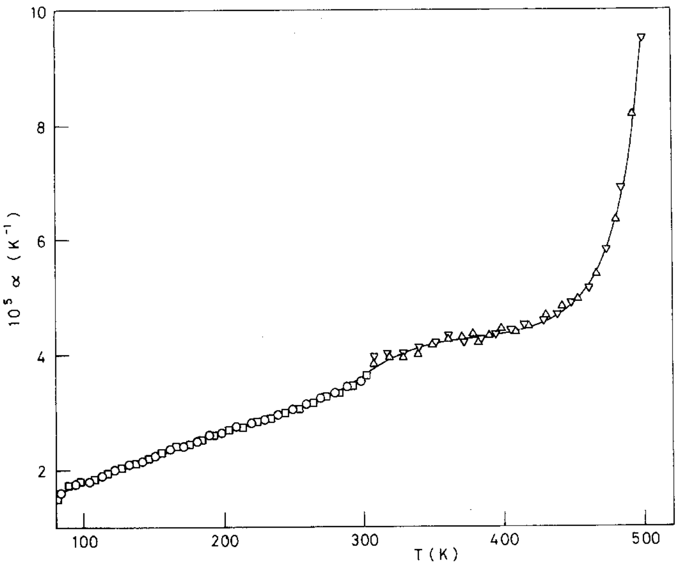

Figure 3 The linear thermal expansion coefficient α of specimens 37: ○ run 1; □ run 2; △ run 3; ▽ run 4.

TABLE II Smoothed values of the linear thermal expansion coefficients α of the specimens described in Table I

T (K)	α for the specimens numbered below (K^{-1})		
	35	36	37
	($\times 10^{-7}$)	($\times 10^{-6}$)	($\times 10^{-5}$)
90	−1.4	4.15	1.70
100	−1.6	4.25	1.79
120	−1.6	4.49	1.95
140	−1.2	4.72	2.12
160	−0.7	5.00	2.28
180	−0.6	5.29	2.46
200	−0.7	5.62	2.63
220	−1.1	5.94	2.79
240	−1.7	6.32	2.97
260	−2.4	6.74	3.14
280	−2.8	7.18	3.32
300	−3.0	7.48	3.63
320	−2.9	7.68	3.94
340	−2.2	7.77	4.12
360	−0.8	7.78	4.23
380	1.7	7.87	4.29
400	5.6	7.98	4.35
420	10.6	8.11	4.50
440	13.3	8.31	4.70
460	5.6	8.78	5.18
480	1.3	9.67	6.33
490	−0.1	10.3	7.62
500	−1.3	11.2	9.50
510	−3.0	12.5	
520	−5.2		

4. Discussion

4.1. Qualitative considerations of the results

The extent to which it is possible to make quantitative predictions about the linear thermal expansion coefficients of a composite structure in terms of the corresponding properties of its constituents is limited by a number of factors. These include: (a) the accuracy to which the constituent properties are known over a range of temperature, (b) the degree of the approximations made by the theories upon which the calculations are based and (c) structural imperfections. Empirical knowledge derived from qualitative comparisons is particularly valuable to design procedure in such cases.

Looking first at the results for specimens 35, (Fig. 1) it may be concluded on the basis of the previous work that the average value of the linear thermal expansion coefficient in the 0° direction between approximately 90 K and 400 K could be raised to zero (a) by a redistribution of laminae between the three orientations, decreasing the proportion in the 0° direction; (b) by using laminae containing fibres parallel to the 0° direction which had a lower fibre volume fraction than those employed in the ±45° directions; (c) by increasing the ±45° inter-ply angles by a few degrees.

Some understanding of the results for specimens 35 may be gained by comparing them with the results for the fibre direction of the corresponding Code 69/HTS unidirectional specimens 26 examined earlier [3], along with the in-plane specimens 28 cut from the 45° direction of the corresponding 90° cross-plied laminate, since the construction of specimens 35 bears a resemblance to a combination of these earlier specimens. The temperature at which the high modulus of the fibres allows them to dominate the behaviour when signs of resin softening occur is very similar for specimens 26 and 35. The position of the minimum somewhat above room temperature is also similar in the two sets of results, when the increasing expansion of the composite with temperature is believed to be due to the influence of the resin. The stress set up in a unidirectional composite by the included fibres appears to enhance departures from the monotonic temperature dependence of the linear thermal expansion coefficient in a direction parallel to the fibres, these departures arising presumably from molecular rearrangements within the resin. This enhancement is not found in the in-plane results for the corresponding 90° cross-plied laminate however. This was apparent in specimens based upon both DLS 351/$BF_3$400 resin (specimens 21 [2] and 33 [3]) and Code 69 resin (specimens 26 [3] and 28 [3]). The absence of a minimum in the results for specimens 35 in the neighbourhood of 200 K, corresponding to that in the results for specimens 26, must therefore be attributed to the influence of the fibres in the ±45° directions. Finally it may be noted that the average value of the results for specimens 35 lies between those for specimens 26 and 28, as expected, and that it is much closer to that for specimens 26, principally because of the predominating influence of the modulus of the fibres in the direction parallel to their length.

Turning to the results for specimens 36, (Fig. 2) corresponding to the 90° direction of bar 17, it is appropriate to compare these with the results for the direction perpendicular to the fibres of the corresponding unidirectional laminate, specimens 27, and with the in-plane results for specimens 28 cut from the 45° direction of the corresponding 90° cross-plied laminate, both of which were investigated earlier [3]. The tridirectional specimens 36 resemble a combination of these unidirectional and bidirectional specimens and, as might be expected, the results lie between the results for these earlier specimens both in magnitude and in temperature dependence until resin softening effects become particularly apparent. At these higher temperatures the influence of the transverse expansion of the 0° component of the assembly is sufficient to cause the linear thermal expansion coefficient to increase with increasing temperature, in contrast to the results for specimens 28. The results also contain a small maximum, centred around 300 K to 320 K, the position of which corresponds to that of the minimum in the results for specimens 35.

Finally the results for specimens 37 (Fig. 3) corresponding to the direction perpendicular to the plane of the laminate may be seen to be basically similar to those for the direction perpendicular to the fibres of the corresponding unidirectional laminate, i.e. specimens 27, but to rise more rapidly at the higher temperatures under the influence of the ±45° components of the assembly. The rise, which becomes particularly rapid as the softening point of the resin is approached, is presumably to be associated with the corresponding fall observed in the 0° direction of the laminate, i.e. specimens 35. The existence of a departure from monotonic behaviour above room temperature may be observed. The origin of this remains uncertain at the present time but its position with respect to temperature resembles those of the minimum in the results for specimens 35 and the maximum in the results for specimens 36.

4.2. Quantitative considerations of the results

As a first step towards correlating the in-plane thermal expansion results with predictions based upon laminate theory [4] it was necessary to derive values for the principal linear thermal expansion coefficients and elastic constants of the constituent laminae. The principal linear thermal expansion coefficients were calculated in the following way. The theoretical models described earlier [2] were applied to the experimental results for the unidirectional specimens 26 [3] to deduce an average value for the linear thermal expansion coefficient of the fibres in a direction parallel to their length, $\alpha_{\parallel}^{f} = -9.8 \times 10^{-7}\ K^{-1}$. An average value for the transverse direction of the fibres, $\alpha_{\perp}^{f} = 1.1 \times 10^{-5}\ K^{-1}$, was deduced in a similar manner, employing the results for specimens 27 [3]. These sets of specimens both came from bar 13 [3], a unidirectional bar based upon the same resin and containing the same type of fibre as the constituent laminae of the present bar 17. It may be noted that these values of $\alpha_{\parallel}^{f}$ and $\alpha_{\perp}^{f}$ lie within the ranges established in previous work [2]. Values for the principal linear thermal expansion coefficients of the constituent laminae of bar 17 were then calculated by substituting these values for $\alpha_{\parallel}^{f}$ and $\alpha_{\perp}^{f}$ back into the equations just employed. In this way a correction was applied for the difference in fibre volume fraction between bar 13 (58.9%) and bar 17 (56.0%). In these calculations and in calculations of the elastic constants of the laminae [1], the experimental result for the linear thermal expansion coefficient of the resin [3] was used in association with a value for the Young's modulus of the fibre (250 GN m^{-2}) based upon a series of measurements made in different ways by the suppliers. Values for the Young's modulus (3.97 GN m^{-2}) and Poisson's ratio (0.41) of the resin were determined at The Railway Technical Centre, Derby. Any influence of voids was ignored in these and subsequent calculations, in which the in-plane response of the 0° ± 45° laminate was calculated employing laminate theory as described earlier [1]. The values calculated for the room temperature linear thermal expansion coefficients for the 0° and 90° directions of the tridirectional laminate were $-3.3 \times 10^{-7}\,\mathrm{K}^{-1}$ and $5.3 \times 10^{-6}\,\mathrm{K}^{-1}$, respectively. Reference to Fig. 1 shows that the value calculated for the 0° direction is in particularly good agreement with the corresponding smoothed experimental data. The corresponding degree of agreement for the 90° direction is as good as can reasonably be expected considering the structural imperfections which always occur in practice, together with the unavoidable assumptions and approximations made during the course of the calculations.

The formal equations upon which a calculation of the out-of-plane linear thermal expansion coefficient could, in principle, be calculated, have been set up [5]. Considering the complexity of the mathematical procedure in relation to the incompleteness of the available data, however, the comparison between theory and experiment afforded by such a calculation would be of limited value at the present time.

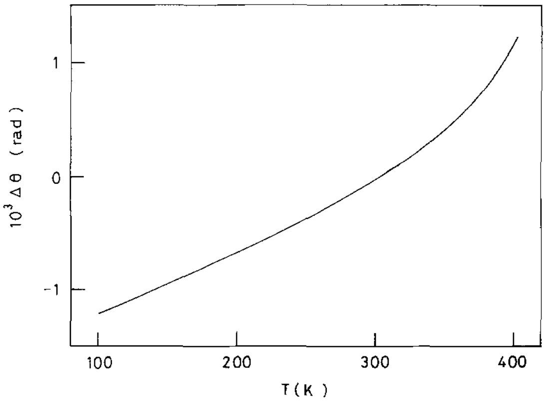

Figure 4 The temperature dependence of the inter-ply angle of a 63° cross-plied laminate of CFRP (bar 4). In this graph $\Delta\theta = \theta_T - \theta_{300}$, where θ_T and θ_{300} are the values of half the acute inter-ply angle at absolute temperatures T and 300 K respectively.

4.3. Temperature dependence of the inter-ply angle of a bidirectional laminate

Since angle-ply laminates form the theme of this paper it is appropriate to note here that the temperature dependence of the dimensions of such a body generally arises from a combination of the thermal expansion and the effects resulting from changes in the angles between the principal fibre directions. The existence of such angular changes, which must arise from the complex stresses to which the components of multidirectional composites are subjected, was indicated by some of the results for the bidirectional specimens studied earlier which had inter-ply angles less than 90°. The case of specimens 8 and 9 [1] may be taken to illustrate this point. It may be recalled that specimens 8 were cut so that their principal axes bisected the acute angle of a 63° cross-plied laminate, while specimens 9 were cut from the same laminate in the direction bisecting the obtuse angle, i.e. at right angles to the direction of specimens 8. The fact that the room temperature linear thermal expansion coefficient of specimens 8 was more negative than that of well-formed pyrolytic graphite [6] supported the suspicion of temperature-dependent angular changes between the principal fibre directions. Employing the small-angle approximation $\tan\Delta\theta \approx \Delta\theta$, where $\Delta\theta$ is the increase in the half-angle θ accompanying a rise in temperature, it is not difficult to show that in the present case

$$\Delta\theta = \frac{\tan\theta_T - 0.6128}{1 + 0.6128\tan\theta_T}$$

$$\text{where } \tan\theta_T = \tan\theta_{300}\left[\frac{1 + \int_{300}^{T}\alpha_9\,\mathrm{d}T}{1 + \int_{300}^{T}\alpha_8\,\mathrm{d}T}\right]$$

θ_T being the value of θ at absolute temperature T consequent upon the shear distortions set up by the mutual restraint of the laminae within the bar [7], and α_8, α_9 the linear thermal expansion coefficients of specimens 8 and 9, respectively. Applying the experimental data for these specimens produced the results displayed in Fig. 4. The asymmetry of the constructions of bars 7, 15 and 17 implies that temperature-dependent shape changes must occur within the planes of specimens prepared from these bars also.

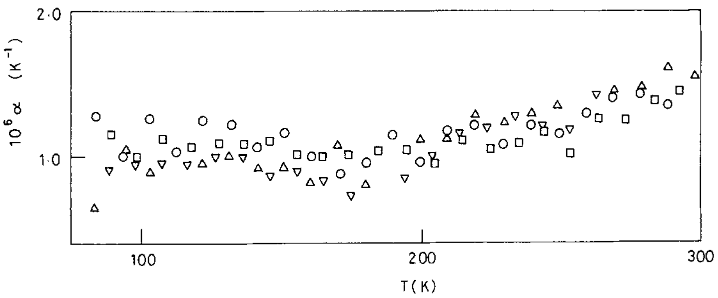

Figure 5 The linear thermal expansion coefficient α of a 90° cross-plied laminate; specimens with principal axis parallel to one fibre direction: dome-shaped ends △ run 1, ▽ run 2: 45° tapered ends ○ run 1, □ run 2.

4.4. Edge effects

If the edge of a CFRP laminate is not cut at right angles to the faces, the weightings of the contributions from the constituent laminae to the thermal expansion of the assembly will in general not be equal. In order to get some idea of the magnitude of this effect a set of specimens measuring 10 mm long × 5 mm × 5 mm was cut from a 90° cross-plied laminate, so that their principal axes ran parallel to one of the principal fibre directions of the laminate. After filing the ends to domes of large curvature the thermal expansion was investigated experimentally below room temperature, in which region there are no complications from resin softening effects. The ends of the specimens were then deliberately filed to a 45° taper, following which the measurements were repeated.

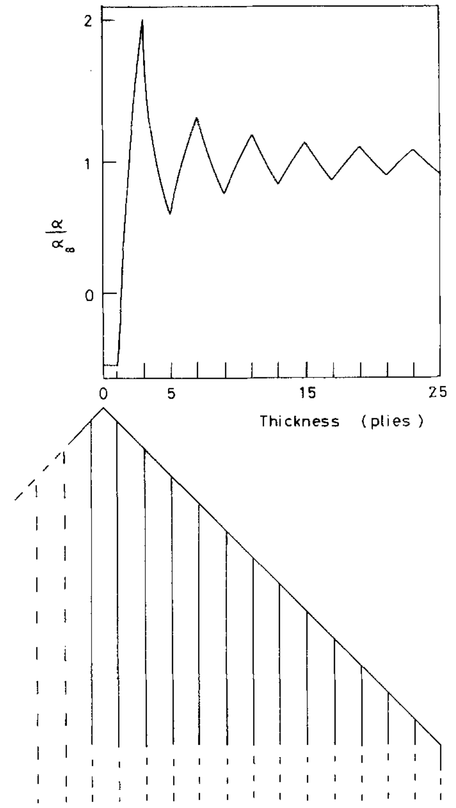

Figure 6 The calculated linear thermal expansion coefficient α of hypothetical slices of CFRP cut perpendicular to the direction of the fibres in the central layer of a 90° cross-plied laminate conforming to the specifications described in the text, in a direction parallel to the fibres in the central layer. The results are expressed in reduced form, in which α_∞ is the hypothetical value of α corresponding to an infinite number of plies.

The two sets of results are compared in Fig. 5, from which it may be seen that any differences lie within the limits of experimental uncertainty. In order to investigate the extent to which this agreement was fortuitous, the situation was then examined analytically, applying experimental data for the corresponding unidirectional laminae in laminate theory to calculate the linear thermal expansion coefficient in a direction parallel to the fibres of the central layer of a 25-layer laminate consisting of fibres arranged in a 0°–90° disposition, for a series of hypothetical slices containing 1, 3, 5, . . . , 25 layers, cut perpendicular to this direction. The results are displayed in Fig. 6, from which it is clear that appropriate account should be taken of effects associated with edges, which depart significantly from the standard form in small blocks of CFRP. Such effects may be safely ignored in structures having the dimensions encountered in typical engineering applications however, because of the smaller fraction of the total length taken up by the edges in such cases.

5. Conclusions

It has been shown how experience gained from the response of the dimensions of simple CFRP structures to temperature change has enabled the behaviour of progressively more complex structures to be predicted with some degree of confidence. In common with earlier findings for simpler structures, the limitation to quantitative understanding is set by a combination of analytical approximations and constructional imperfections. One may finally conclude that the design of CFRP structures possessing required directional thermal expansion characteristics may best be achieved by deploying empirical knowledge supplemented by numerical analysis where this is possible. When the tolerances are small however, the thermal expansion characteristics of the structure should finally be checked by direct experimental measurement.

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